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Three Inequivalent Mass-Degenerate Majorana Neutrinos and a Model of Their Splitting for Neutrino Oscillations

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Abstract

The mass matrix of three Majorana neutrinos of equal mass is not necessarily proportional to the identity matrix, but expressible in terms of two angles and one phase. We discuss how such a mass matrix may be stable or unstable against radiative corrections. We then propose a model with additional explicit breaking of the threefold degeneracy to account for the atmospheric neutrino data, while the radiative breaking explains the solar neutrino data, using the large-angle Mikheyev-Smirnov-Wolfenstein solution. Our model requires a nonzero effective ν_e mass for neutrinoless double beta decay close to the present experimental upper limit of 0.2 eV.

There are now several experimental results[1, 2, 3] favoring neutrino oscillations as their explanation. Since only the differences of the squares of neutrino masses are involved, there has always been a lot of theoretical interest in considering nearly degenerate neutrino masses[4]. The origin of their splitting may in fact be radiative[5] and some simple specific models have been proposed[6]. In the case of 3 Majorana neutrinos, it has been shown[7] that all may be identical in mass and yet are inequivalent to one another because their mass matrix may contain 2 angles and 1 phase which cannot be redefined away. In the following we will discuss the stability of this construction under radiative corrections.

We will then propose a model with additional explicit breaking of the threefold degeneracy to account for the atmospheric neutrino data[1], while the radiative breaking explains the solar neutrino data[2], using the large-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution[8]. Our model assumes all 3 neutrino masses to be of order 0.5 eV, to be consistent[9] with their possible role in cosmic structure formation, in the light of recent astrophysical evidence[10] favoring a nonzero cosmological constant. An effective ν_e mass for neutrinoless double beta decay close to the present experimental upper limit[11] of 0.2 eV is also required.

We start out with the most general 3×3 unitary matrix linking $(\nu_e, \nu_\mu, \nu_\tau)$ with their mass eigenstates (ν_1, ν_2, ν_3), i.e.[12]

$$U = \left[\begin{array}{ccc} c_1 c_3 & s_1 c_3 e^{-i\delta_1} & s_3 e^{-i\delta_2} \\ -s_1 c_2 e^{i\delta_1} - c_1 s_2 s_3 e^{i(\delta_2+\delta_3)} & c_1 c_2 - s_1 s_2 s_3 e^{i(\delta_3+\delta_2-\delta_1)} & s_2 c_3 e^{i\delta_3} \\ s_1 s_2 e^{i(\delta_1-\delta_3)} - c_1 c_2 s_3 e^{i\delta_2} & -c_1 s_2 e^{-i\delta_3} - s_1 c_2 s_3 e^{i(\delta_2-\delta_1)} & c_2 c_3 \end{array} \right], \qquad (1)$$

which has 3 angles and 3 phases. Let us set $s_3 = 0$ ($c_3 = 1$), $\delta_1 = \pi/2$, $\delta_2 \equiv \delta$, and $\delta_3 = \pi/2 - \delta$, and multiply on the left by the diagonal matrix $(1, -i, e^{-i\delta})$, then

$$U = \left[\begin{array}{ccc} c_1 & -is_1 & 0 \\ -s_1 c_2 & -ic_1 c_2 & s_2 e^{-i\delta} \\ s_1 s_2 & ic_1 s_2 & c_2 e^{-i\delta} \end{array} \right]. \qquad (2)$$

Since all neutrino masses are assumed equal, the mass matrix in the (ν_1, ν_2, ν_3) basis is

just m times the identity matrix. However, in the $(\nu_e, \nu_\mu, \nu_\tau)$ basis, it is given by

$$\mathcal{M} = mU^*U^\dagger = m\begin{bmatrix} c_0 & -s_0c_2 & s_0s_2 \\ -s_0c_2 & -c_0c_2^2 + s_2^2 e^{2i\delta} & s_2c_2(c_0 + e^{2i\delta}) \\ s_0s_2 & s_2c_2(c_0 + e^{2i\delta}) & -c_0s_2^2 + c_2^2 e^{2i\delta} \end{bmatrix}, \tag{3}$$

where $c_0 \equiv c_1^2 - s_1^2$ and $s_0 \equiv 2s_1c_1$. As shown in Ref.[7], this mass matrix (which has 2 angles and 1 phase) cannot be reduced further and there is CP violation if $e^{2i\delta} \neq \pm 1$. Another way to see this is to realize that U of Eq. (2) cannot be written in the form $U = DO$, where D is a diagonal matrix containing phases and O is an orthogonal matrix.

Even though U of Eq. (2) is nontrivial, the absence of any mass differences among the 3 neutrinos will not result in oscillations. However, in the presence of radiative corrections from the charged leptons, neutrino mass differences will occur. It is easiest to do this in the (ν_1, ν_2, ν_3) basis. Without radiative corrections, this mass matrix is just m times the identity matrix. With radiative corrections coming from the wavefunction renormalization of ν_τ due to m_τ, we find

$$\begin{aligned} \mathcal{M}' &= mU^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+\epsilon \end{bmatrix} U^*U^\dagger \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+\epsilon \end{bmatrix} U \\ &= m\begin{bmatrix} 1 + 2s_1^2s_2^2\epsilon & 0 & 2c_\delta s_1s_2c_2\epsilon \\ 0 & 1 + 2c_1^2s_2^2\epsilon & -2s_\delta c_1s_2c_2\epsilon \\ 2c_\delta s_1s_2c_2\epsilon & -2s_\delta c_1s_2c_2\epsilon & 1 + 2c_2^2\epsilon \end{bmatrix}, \end{aligned} \tag{4}$$

where

$$\epsilon = -\frac{G_F m_\tau^2}{16\pi^2\sqrt{2}} \ln \frac{\Lambda^2}{m_W^2}, \tag{5}$$

with Λ equal to the scale at which the degenerate mass matrix is defined.

The characteristic polynomial equation of $\mathcal{M}'/m$ is given by

$$x^3 - x^2 + x[s_2^2c_2^2(1 - c_\delta^2s_1^2 - s_\delta^2c_1^2) + s_1^2c_1^2s_2^4] = 0, \tag{6}$$

where $x = (\lambda - 1)/2\epsilon$. Hence one eigenvalue λ is 1 and it corresponds to the eigenstate

$$\nu' = N^{-1/2}[c_\delta c_1c_2\nu_1 - s_\delta s_1c_2\nu_2 - s_1c_1s_2\nu_3] = N^{-1/2}[c_2(c_\delta c_1^2 - is_\delta s_1^2)\nu_e - s_1c_1e^{i\delta}\nu_\mu], \tag{7}$$

where $N = c_\delta^2 c_1^2 c_2^2 + s_\delta^2 s_1^2 c_2^2 + s_1^2 c_1^2 s_2^2$. This shows that the original pattern of 2 mixing angles and 1 phase is generally unstable and no matter what values they take, a mass eigenstate exists without ν_τ as given above. For $\mathcal{M}'$ to be stable, the off-diagonal elements must go to zero, but since they depend on the original 2 angles and 1 phase, the condition of stability imposes severe constraints on their values.

To understand the atmospheric neutrino data in terms of oscillations, we need $s_2^2 \sim c_2^2 \sim 0.5$, hence the requirement of stability forces s_1 (as well as s_δ) to be small, which implies $\nu' \simeq \nu_1 \simeq \nu_e$. This means that we must take the small-angle MSW solution for the solar neutrino data. However from Eq. (6), it can be shown that the mass eigenvalues of the other 2 states are both smaller than 1 because ϵ of Eq. (5) is negative. Thus there cannot be any resonance enhancement of oscillations from ν_e interactions in the sun and this scheme does not work.

We now propose to break the threefold degeneracy also explicitly by a mass term

$$m'(s'\nu_2 + c'\nu_3)^2, \tag{8}$$

where m' is chosen such that $(m+m')^2 - m^2$ is of order 10^{-3} eV^2 to be suitable for atmospheric neutrino oscillations. Then the 2×2 mass submatrix spanning ν_1 and $c'\nu_2 - s'\nu_3$ is given by

$$\mathcal{M}'' = m \begin{bmatrix} 1 + 2s_1^2 s_2^2 \epsilon & -2s' c_\delta s_1 s_2 c_2 \epsilon \\ -2s' c_\delta s_1 s_2 c_2 \epsilon & 1 + 2(c'^2 c_1^2 s_2^2 + s'^2 c_2^2 + 2 s_\delta s' c' c_1 s_2 c_2)\epsilon \end{bmatrix}. \tag{9}$$

Consider the simplified case of $s_\delta = -1$ ($c_\delta = 0$), then $\mathcal{M}''$ is diagonal. Hence ν_1 and $c'\nu_2 - s'\nu_3$ are eigenstates with eigenvalues

$$m_1 = m(1 + 2s_1^2 s_2^2 \epsilon), \quad m_2 = m[1 + 2(c'^2 s_2^2 c_1^2 + s'^2 c_2^2 - 2s'c's_2c_2c_1)\epsilon] \tag{10}$$

respectively. We choose the convention that $c_1^2 > 1/2$, so that ν_1 is mostly ν_e. Therefore, the requirement $m_1 < m_2$ means

$$s_1^2 s_2^2 > c'^2 s_2^2 c_1^2 + s'^2 c_2^2 - 2s'c's_2c_2c_1 = (c's_2c_1 - s'c_2)^2, \tag{11}$$

resulting in the following condition on c_1:

$$\frac{1}{\sqrt{2}} < c_1 < \frac{s'c'c_2 + \sqrt{2s_2^2 - s'^2}}{s_2(1+c'^2)}. \tag{12}$$

Note that $s' = 0$ is not a solution.

With the new set of mass eigenstates, the transformation matrix U of Eq. (2) becomes

$$U_{\alpha i} = \left[\begin{array}{ccc} c_1 & -ic's_1 & -is's_1 \\ -s_1c_2 & -i(c'c_1c_2 + s's_2) & -i(s'c_1c_2 - c's_2) \\ s_1s_2 & i(c'c_1s_2 - s'c_2) & i(s'c_1s_2 + c'c_2) \end{array} \right]. \tag{13}$$

Using the well-known expressions for the probabilities of neutrino oscillations, we find in the atmospheric case,

$$P_{\mu\mu} = 1 - 2|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2)(1 - \cos(\Delta m^2 L/2E)) \tag{14}$$

$$P_{ee} = 1 - 2|U_{e3}|^2(1 - |U_{e3}|^2)(1 - \cos(\Delta m^2 L/2E)) \tag{15}$$

$$P_{\mu e} = P_{e\mu} = 2|U_{e3}|^2|U_{\mu 3}|^2(1 - \cos(\Delta m^2 L/2E)), \tag{16}$$

where $\Delta m^2 = (m + m')^2 - m^2$. In the solar case,

$$P_{ee} = 1 - 2|U_{e3}|^2(1 - |U_{e3}|^2) - 2|U_{e1}|^2|U_{e2}|^2(1 - \cos(\Delta' m^2 L/2E)), \tag{17}$$

where $\Delta' m^2 = m_2^2 - m_1^2$. Consider again the small-angle MSW solution of the solar neutrino data. This requires $c_1 \simeq 1$, but then Eq. (11) implies that $(c's_2 - s'c_2)^2 \simeq |U_{\mu 3}|^2$ has to be very small, which is disallowed by the atmospheric neutrino data. Note especially that this restriction is independent of the value of m. Hence only the large-angle MSW solution[13] will be considered from now on.

We allow ϵ of Eq. (5) to be divided by $\cos^2\beta$, where $\tan\beta \equiv v_2/v_1$ in a two-Higgs doublet model. We then fix

$$\Delta' m^2 = m_2^2 - m_1^2 \simeq \frac{4m^2 G_F m_\tau^2}{16\pi^2\sqrt{2}\cos^2\beta} \ln\frac{\Lambda^2}{m_W^2}\left[s_1^2 s_2^2 - (c's_2c_1 - s'c_2)^2\right] \tag{18}$$

to be equal to 10^{-5} eV^2 to fit the solar data. This is necessary because we will set $\Lambda = 10^{14}$ GeV and choose m to be 0.5 eV or less[9] which then require $\cos^2\beta < 1$ to obtain the desired value of $\Delta' m^2$. From the CHOOZ reactor data[14], we require

$$|U_{e3}|^2 < 0.025. \tag{19}$$

From the atmospheric neutrino data[1], we require

$$0.84 < 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = \sin^2 2\theta_{atm} < 1. \tag{20}$$

From the large-angle MSW solution[13] to the solar neutrino data[2], we require

$$0.7 < 4|U_{e1}|^2|U_{e2}|^2 = \sin^2 2\theta_{sol} < 0.9. \tag{21}$$

From neutrinoless double beta decay[11], we require

$$m_{ee} = mc_0 = m(c_1^2 - s_1^2) < 0.2 \text{ eV}. \tag{22}$$

We then choose $m = 0.5$ eV, and scan the parameter space of s', s_1, and s_2 for solutions. We find the following allowed ranges of values (each obtained with the others free):

$$0.16 < s' < 0.29, \quad 0.55 < s_1 < 0.64, \quad 0.66 < s_2 < 0.84. \tag{23}$$

In Table 1 we show some typical solutions. It is clear that the effective neutrino mass m_{ee} for neutrinoless double beta decay is not far below its current experimental upper limit of 0.2 eV and is accessible to the next generation of such experiments[15]. We note also that Eq. (18) depends only on the ratio $m/\cos\beta$. Hence if we decrease m, then the same set of values for s', s_1, and s_2 is a solution if we also decrease $\cos\beta$ by the same factor. Of course, m_{ee} is also reduced, but if m plays a role in cosmic structure formation, then m_{ee} cannot be an order of magnitude smaller than 0.1 eV.

In the above, we have chosen $s_\delta = -1$ in Eq. (9). This allows Eq. (11) to be satisfied with the widest range of parameter values. For a general s_δ, Eq. (11) is replaced by

$$s_1^2 s_2^2 > c'^2 s_2^2 c_1^2 + s'^2 c_2^2 + 2 s_\delta s' c' s_2 c_2 c_1 = (c' s_2 c_1 - s' c_2)^2 + 2(1 + s_\delta) s' c' s_2 c_2 c_1, \tag{24}$$

which is clearly more restrictive.

In conclusion, we have shown in this paper how three inequivalent mass-degenerate Majorana neutrinos are unstable against radiative corrections due to m_τ. However, if we add an explicit mass term which breaks this threefold degeneracy to account for the atmospheric neutrino data, the remaining radiative splitting is able to account for the solar data, but only with the large-angle MSW solution, resulting in an effective neutrino mass for neutrinoless double beta decay close to the present experimental upper limit of 0.2 eV.

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s'	s_1	s_2	$\cos\beta$	$\sin^2 2\theta_{atm}$	$\sin^2 2\theta_{sol}$	m_{ee}
0.18	0.60	0.68	0.06	0.87	0.89	0.14 eV
0.23	0.58	0.72	0.07	0.88	0.85	0.16 eV
0.23	0.62	0.72	0.21	0.89	0.90	0.12 eV
0.23	0.62	0.76	0.18	0.95	0.90	0.12 eV
0.23	0.62	0.80	0.14	0.99	0.90	0.12 eV
0.28	0.56	0.74	0.10	0.85	0.79	0.19 eV

Table 1: Typical allowed parameter values of this model for $m = 0.5$ eV.